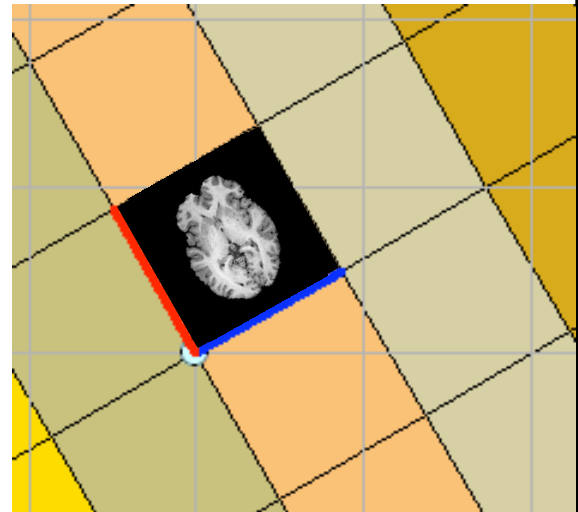
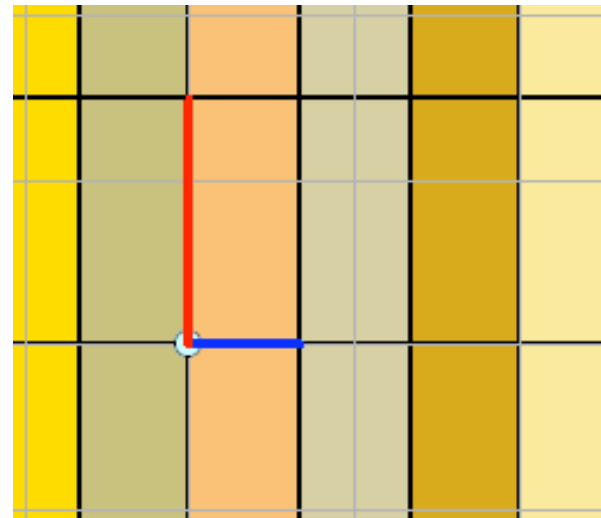
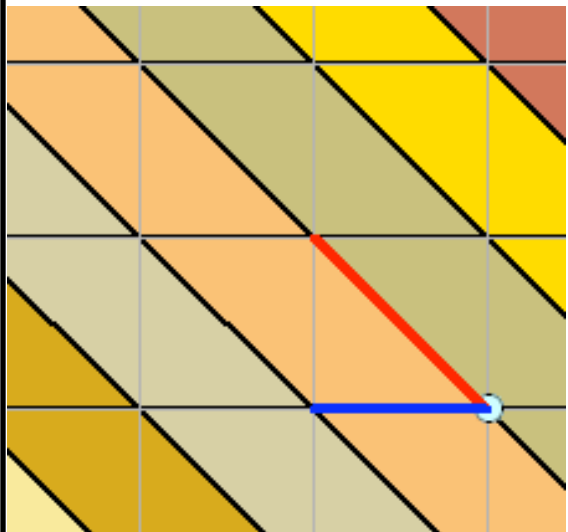
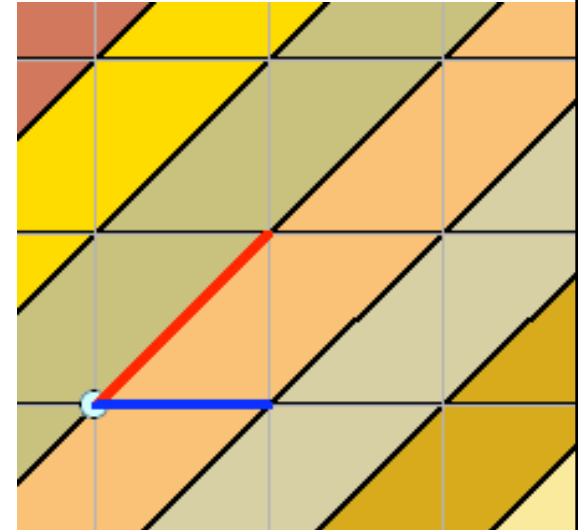
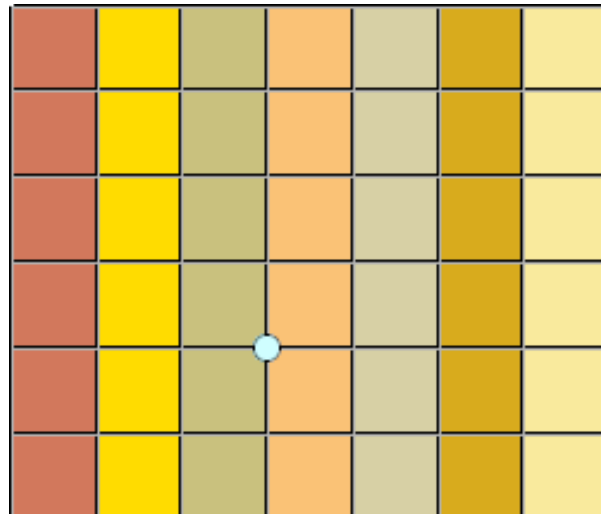
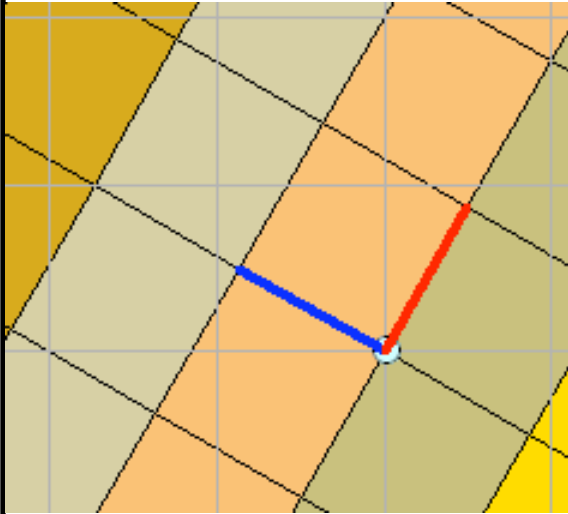
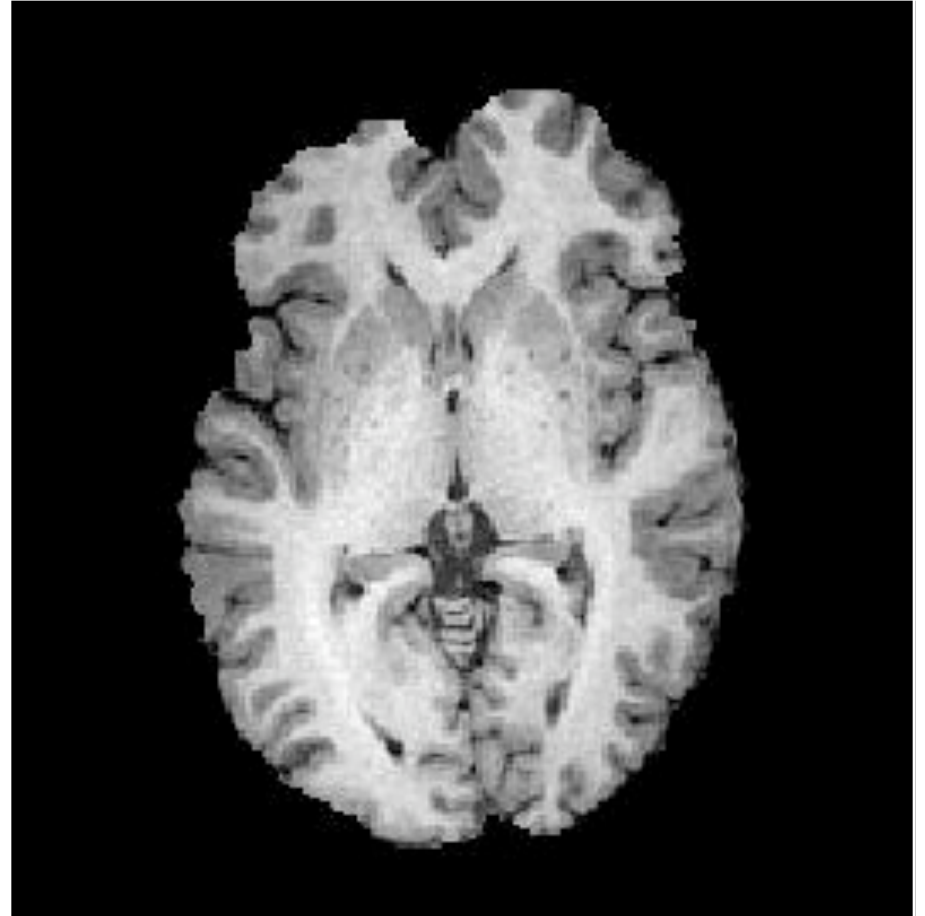


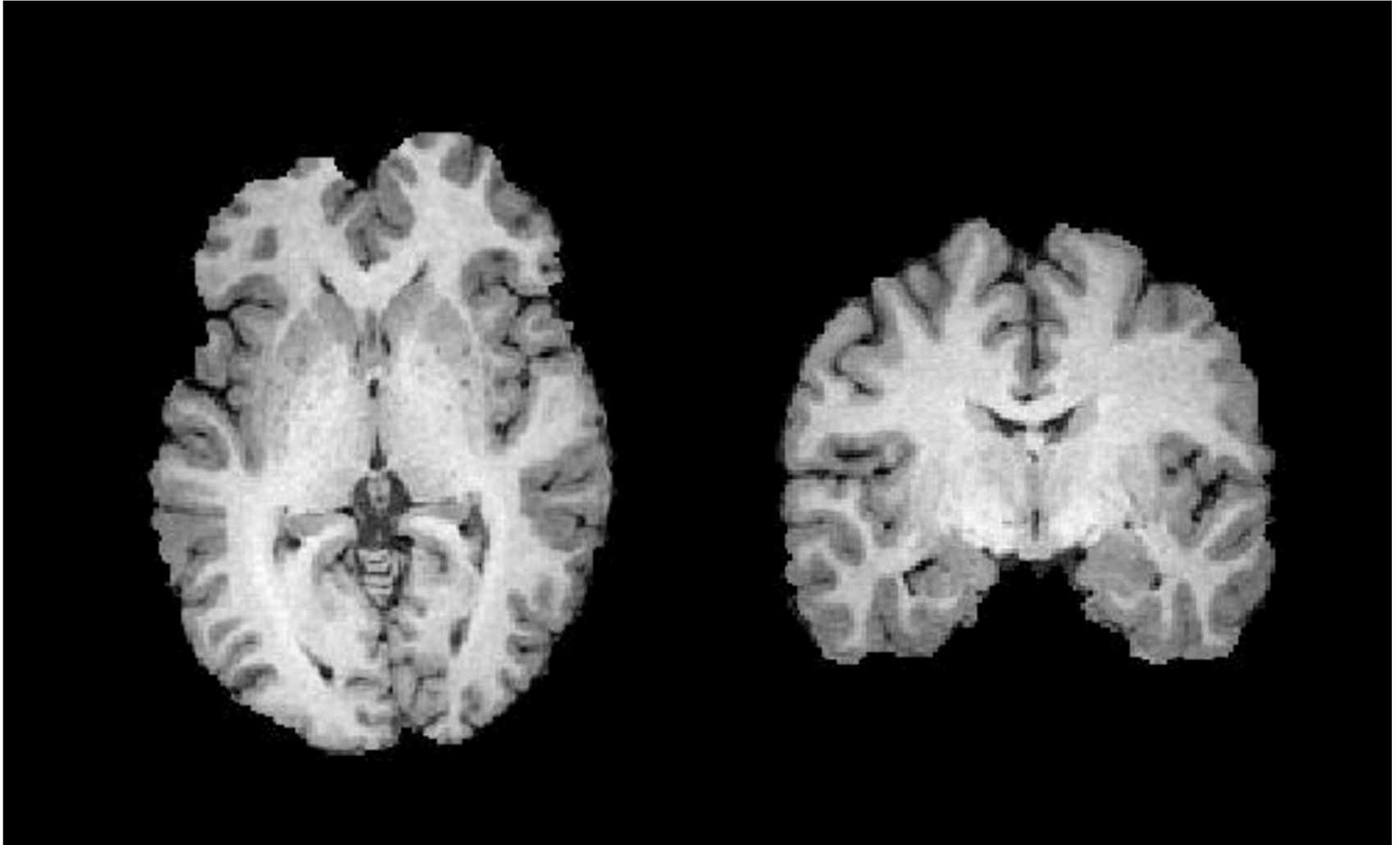
Image Processing and Registration



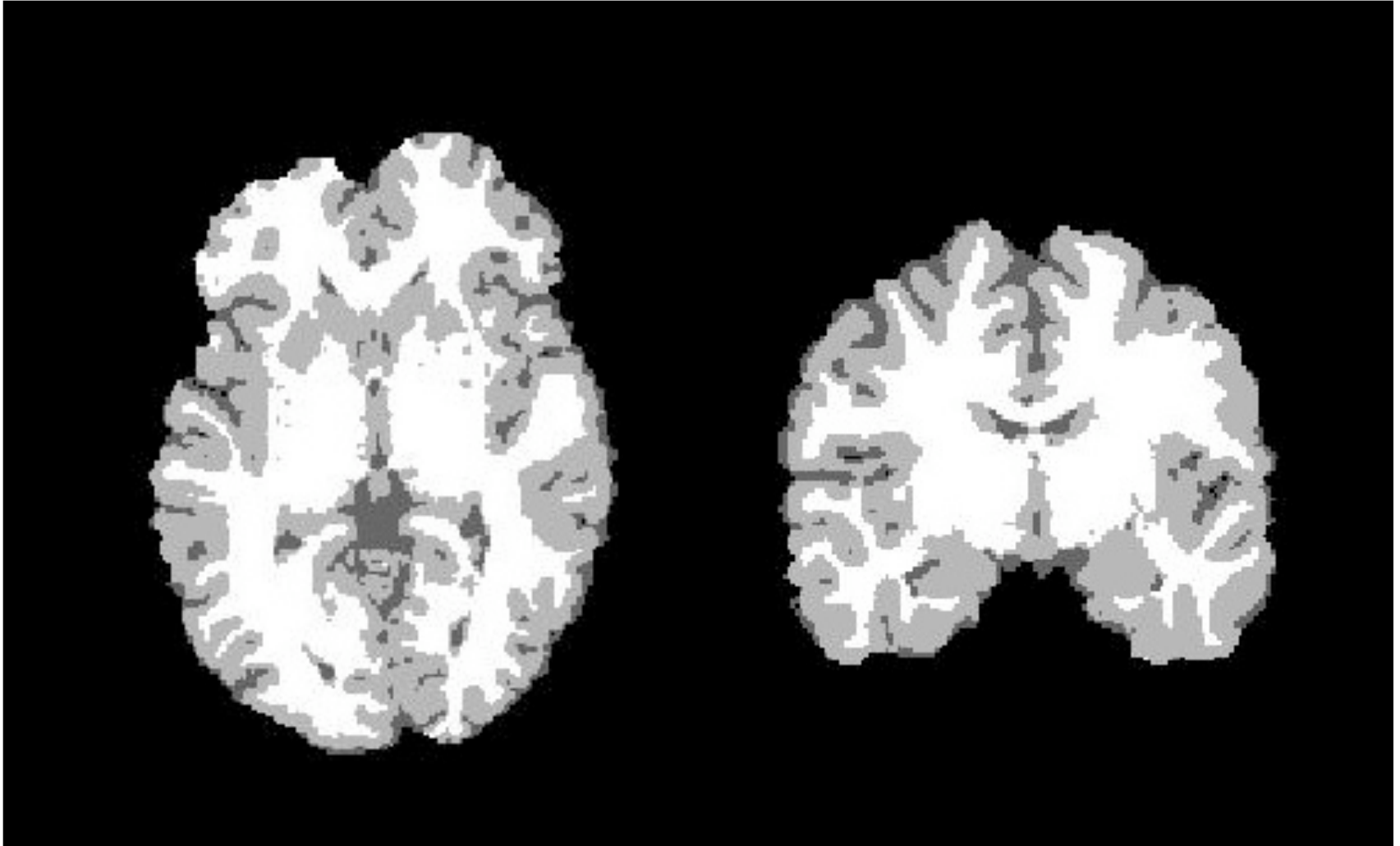
Re: k-space to image



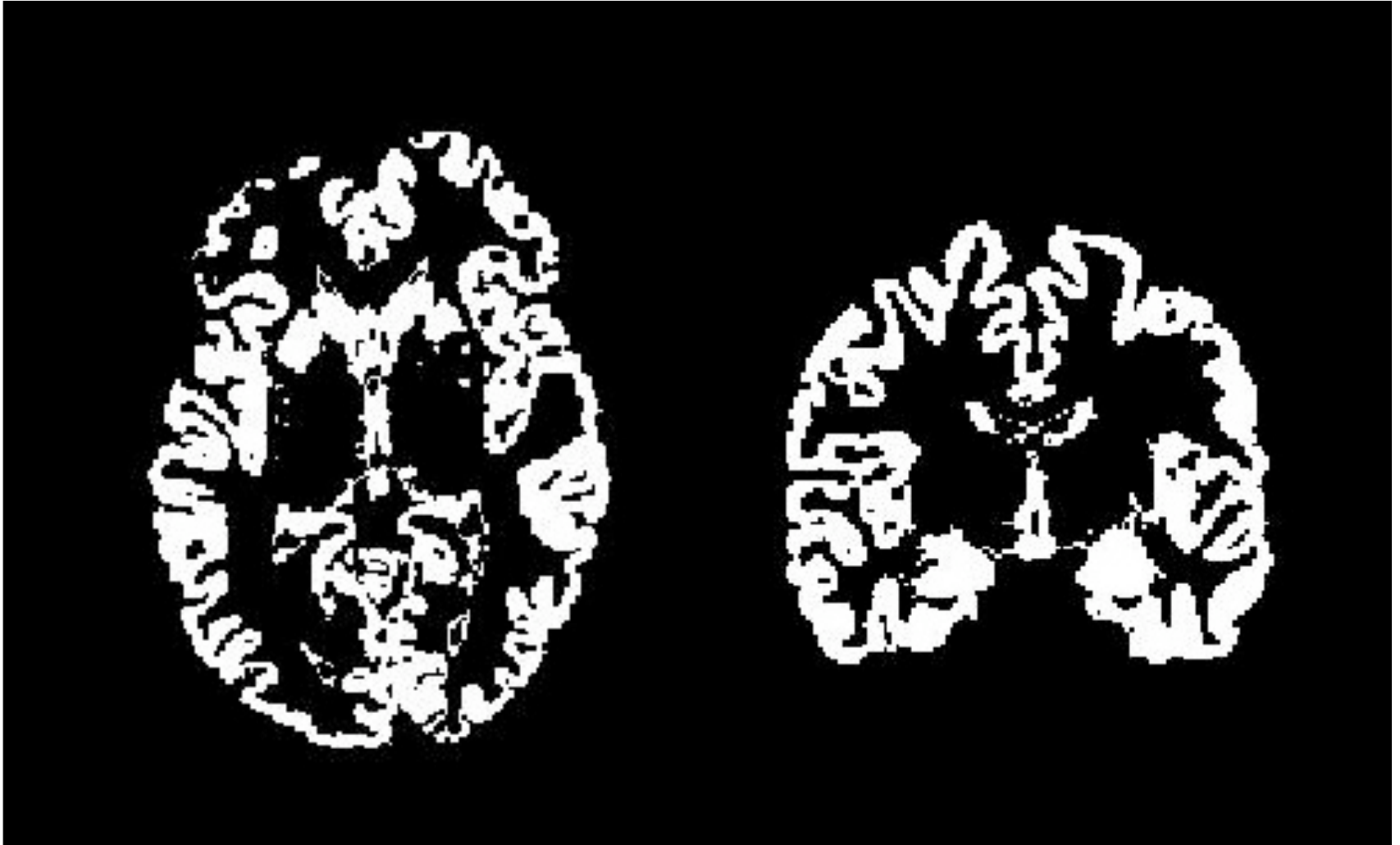
skull-stripping



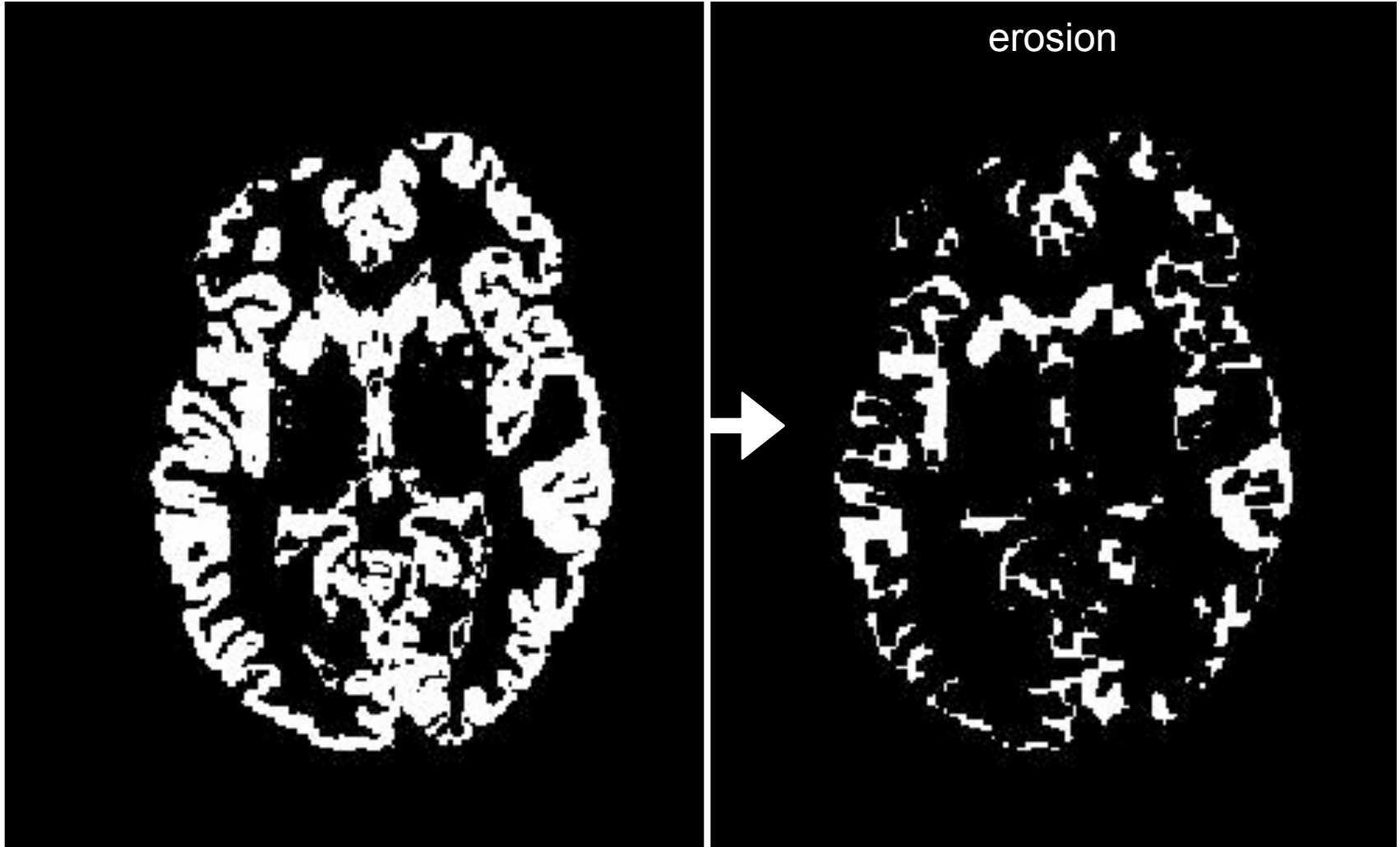
tissue-class segmentation (+ inhomogeneity correction)



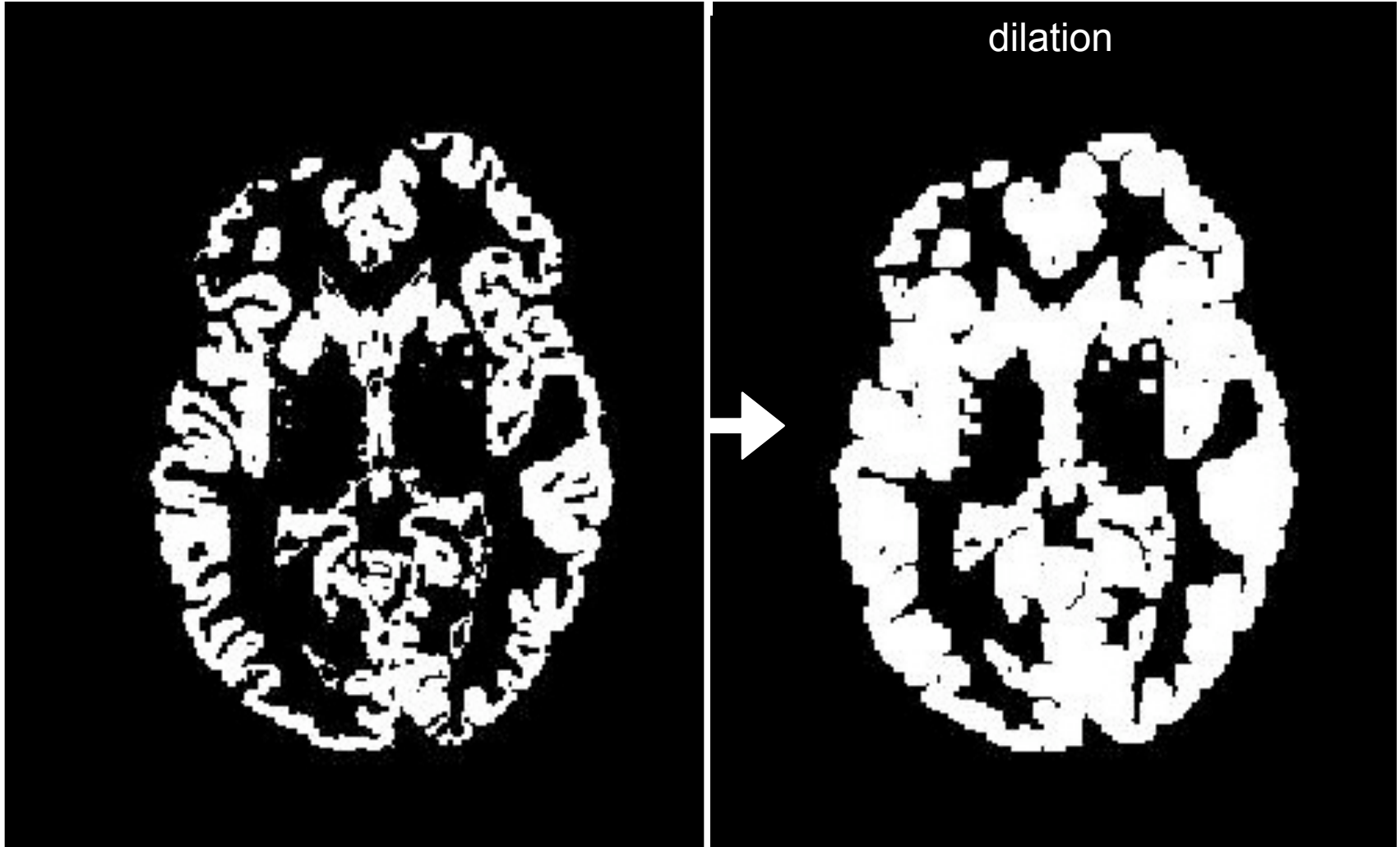
tissue-class segmentation



morphological image processing



morphological image processing



morphological image processing

Dilation example

maximum value in neighborhood

structuring element



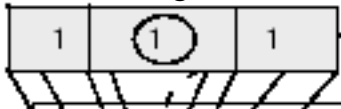
1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

input image

1	1
1	
0	
0	
0	

output image

structuring element



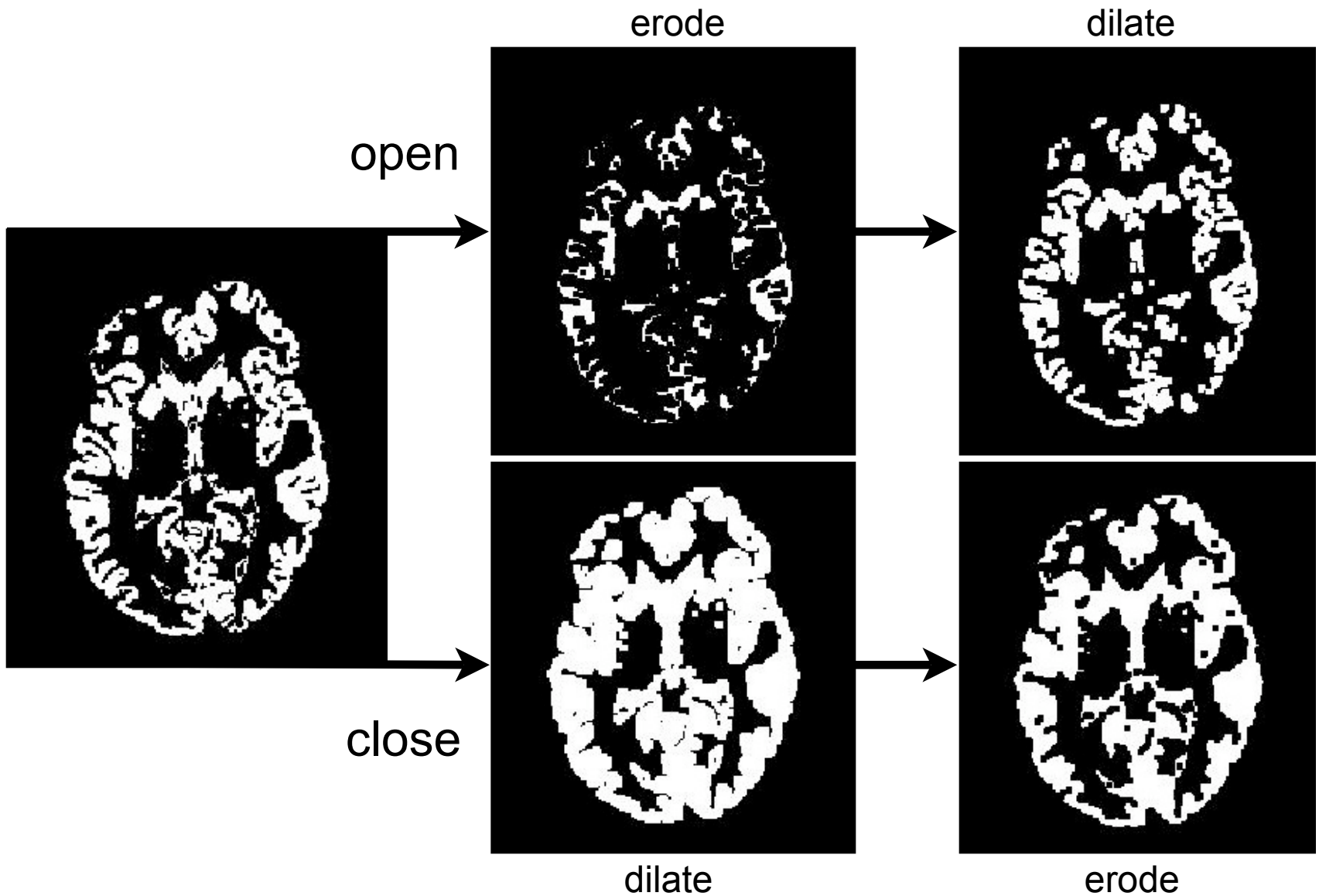
16	14	14	17	19	15	21
53	57	61	62	64	60	68
126	128	124	122	125	125	127
132	130	133	132	131	132	130
140	138	137	143	138	137	134
143	141	138	142	140	134	144
138	142	137	139	138	132	136

input image

16	16
57	
128	
132	
140	
143	
142	

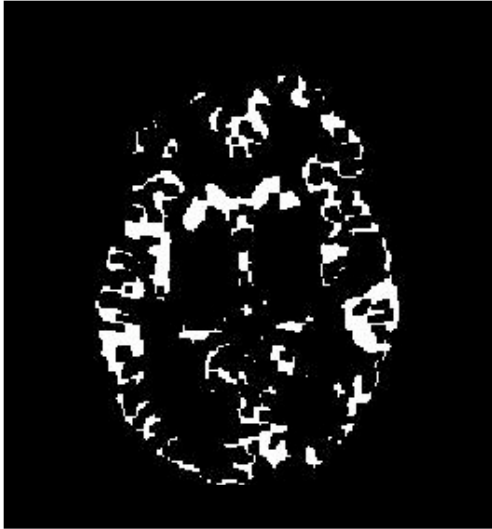
output image

morphological image processing



morphological image processing

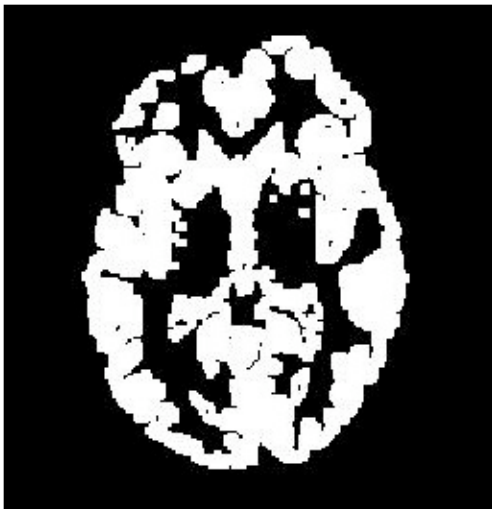
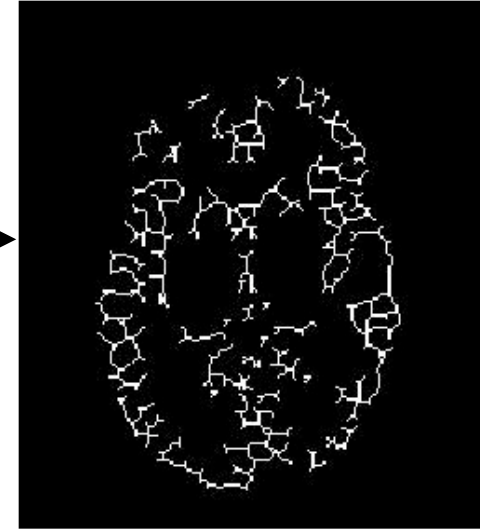
erode



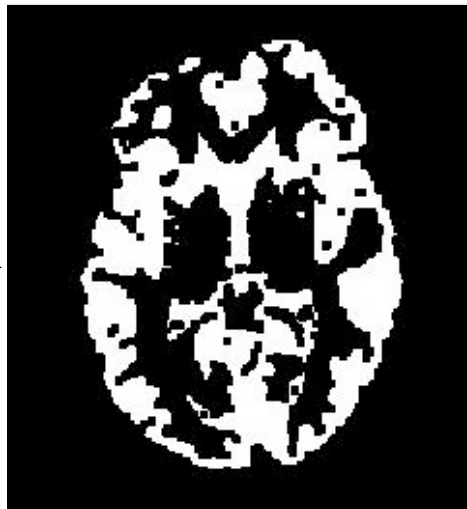
dilate



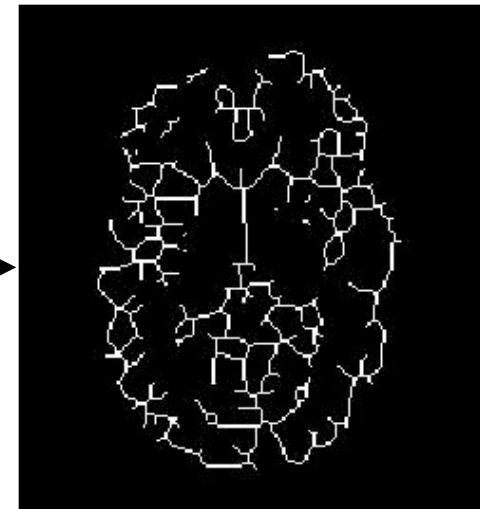
open skeleton



dilate



erode



closed skeleton

morphological reconstruction

Morphological reconstruction:

- Processing is based on two images, a marker and a mask, rather than one image and a structuring element.
- Processing repeats until stability (the image no longer changes).
- Processing is based on the concept of connectivity, rather than a structuring element

Examples:

1. pixel connectivity
2. flood-fill operations
3. peaks and valleys

```
0 0 0 0 0 0
0 1 1 0 0 0
0 1 1 0 0 0
0 0 0 1 1 0
0 0 0 1 1 0
```

pixel connectivity

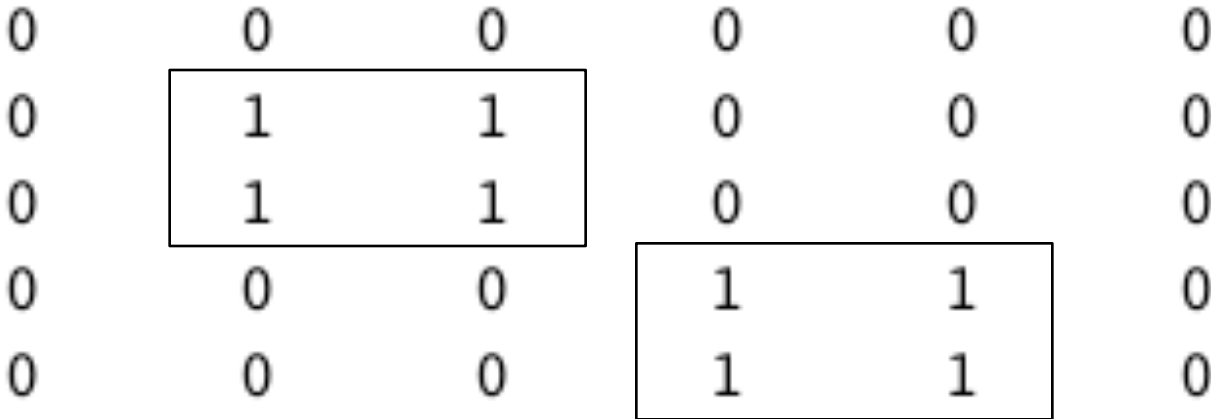
```
0 0 0 0 0 0
0 1 1 0 0 0
0 1 1 0 0 0
0 0 0 1 1 0
0 0 0 1 1 0
```

```
0 0 0 0 0 0
0 1 1 0 0 0
0 1 1 0 0 0
0 0 0 1 1 0
0 0 0 1 1 0
```

pixel connectivity

0	0	0	0	0	0
0	1	1	0	0	0
0	1	1	0	0	0
0	0	0	1	1	0
0	0	0	1	1	0

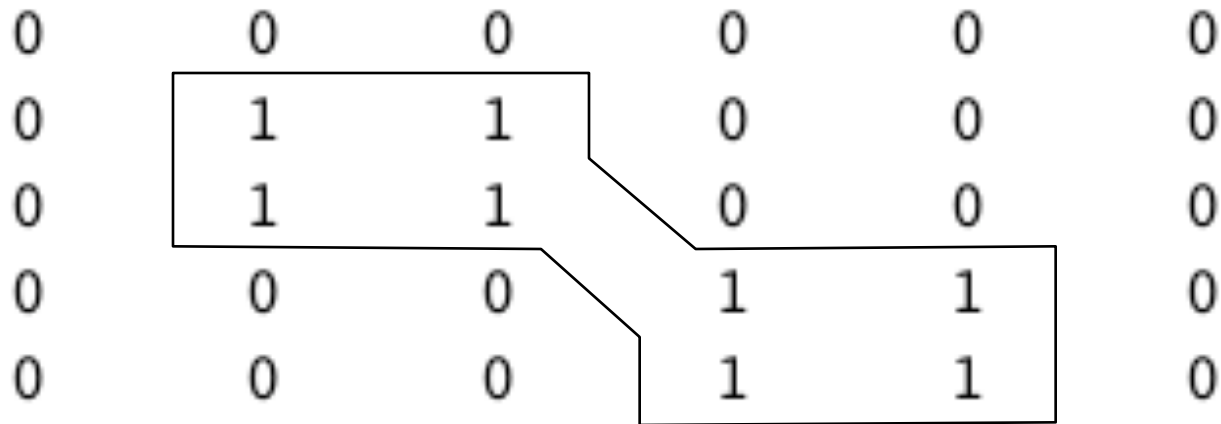
0	0	0	0	0	0
0	1	1	0	0	0
0	1	1	0	0	0
0	0	0	1	1	0
0	0	0	1	1	0



```
0 0 0 0 0 0
0 1 1 0 0 0
0 1 1 0 0 0
0 0 0 1 1 0
0 0 0 1 1 0
```

pixel connectivity

```
0 0 0 0 0 0
0 1 1 0 0 0
0 1 1 0 0 0
0 0 0 1 1 0
0 0 0 1 1 0
```



```

0 0 0 0 0 0
0 1 1 0 0 0
0 1 1 0 0 0
0 0 0 1 1 0
0 0 0 1 1 0

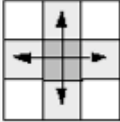
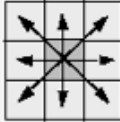

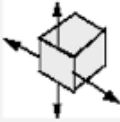
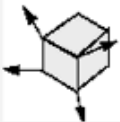
```

pixel connectivity

```

0 0 0 0 0 0
0 1 1 0 0 0
0 1 1 0 0 0
0 0 0 1 1 0
0 0 0 1 1 0

```

Two-Dimensional Connectivities		
4-connected	<p>Pixels are connected if their edges touch. This means that a pair of adjoining pixels are part of the same object only if they are both on and are connected along the horizontal or vertical direction.</p>	
8-connected	<p>Pixels are connected if their edges or corners touch. This means that if two adjoining pixels are on, they are part of the same object, regardless of whether they are connected along the horizontal, vertical, or diagonal direction.</p>	
Three-Dimensional Connectivities		
6-connected	<p>Pixels are connected if their faces touch.</p>	 <p>6 faces</p>
18-connected	<p>Pixels are connected if their faces or edges touch.</p>	 <p>6 faces + 12 edges</p>
26-connected	<p>Pixels are connected if their faces, edges, or corners touch.</p>	 <p>6 faces + 12 edges + 8 corners</p>

```

0 0 0 0 0 0 0 0
0 1 1 1 1 1 0 0
0 1 0 0 0 1 0 0
0 1 0 0 0 1 0 0
0 1 0 0 0 1 0 0
0 1 1 1 1 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0

```

flood filling

```

0 0 0 0 0 0 0 0
0 1 1 1 1 1 0 0
0 1 1 1 1 1 0 0
0 1 1 1 1 1 0 0
0 1 1 1 1 1 0 0
0 1 1 1 1 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0

```

```

0 0 0 0 0 0 0 0
0 1 1 1 1 1 0 0
0 1 0 0 0 1 0 0
0 1 0 0 0 1 0 0
0 1 0 0 0 1 0 0
0 1 1 1 1 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0

```

```

0 0 0 0 0 0 0 0
0 1 1 1 1 1 0 0
0 1 0 0 0 1 0 0
0 1 0 0 0 1 0 0
0 1 0 0 0 1 0 0
0 1 1 1 1 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0

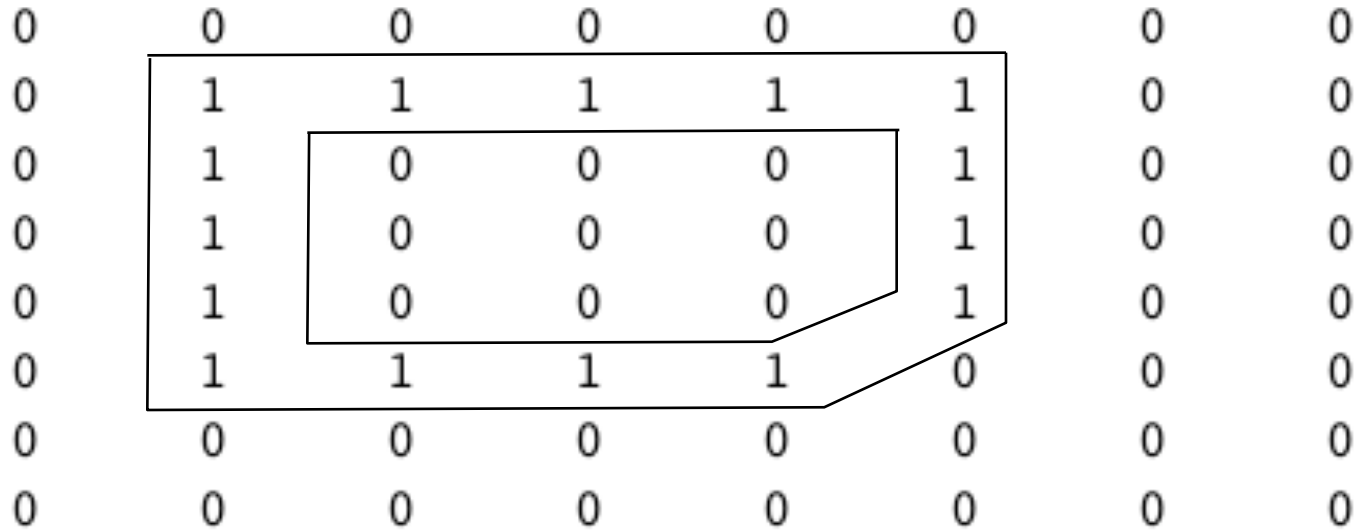
```

flood filling

```

0 0 0 0 0 0 0 0
0 1 1 1 1 1 0 0
0 1 1 1 1 1 0 0
0 1 1 1 1 1 0 0
0 1 1 1 1 1 0 0
0 1 1 1 1 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0

```



```

0 0 0 0 0 0 0 0
0 1 1 1 1 1 0 0
0 1 0 0 0 1 0 0
0 1 0 0 0 1 0 0
0 1 0 0 0 1 0 0
0 1 1 1 1 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0

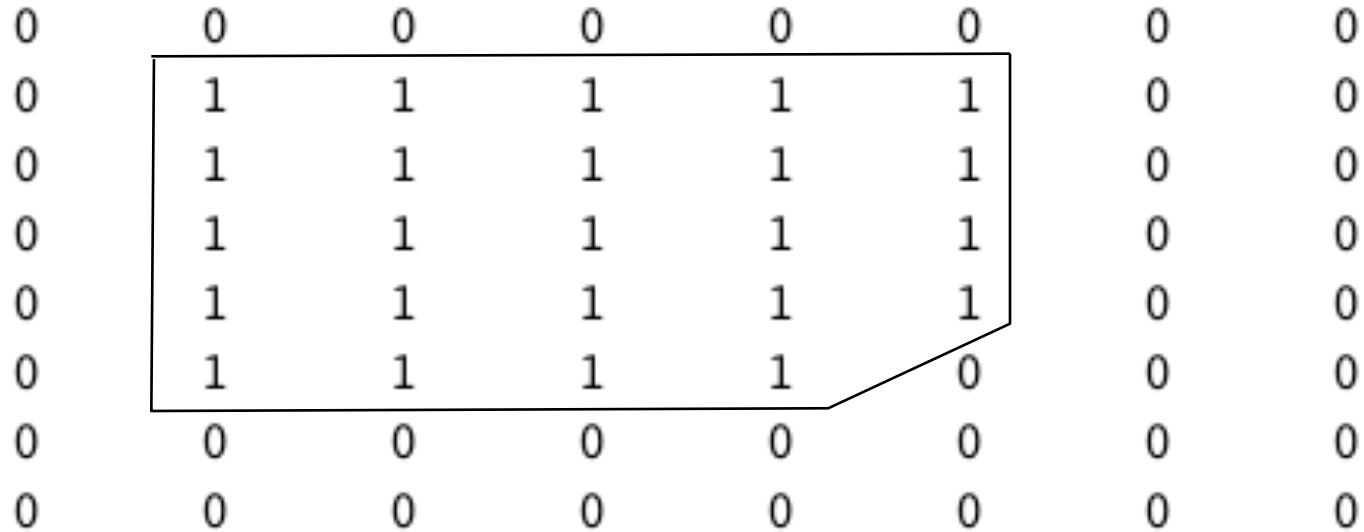
```

flood filling

```

0 0 0 0 0 0 0 0
0 1 1 1 1 1 0 0
0 1 1 1 1 1 0 0
0 1 1 1 1 1 0 0
0 1 1 1 1 1 0 0
0 1 1 1 1 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0

```



10	10	10	10	10	10	10	10	10	10
10	13	13	13	10	10	11	10	11	10
10	13	13	13	10	10	10	11	10	10
10	13	13	13	10	10	11	10	11	10
10	10	10	10	10	10	10	10	10	10
10	11	10	10	10	18	18	18	10	10
10	10	10	11	10	18	18	18	10	10
10	10	11	10	10	18	18	18	10	10
10	11	10	11	10	10	10	10	10	10
10	10	10	10	10	10	11	10	10	10

peaks & valleys

0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0
0	0	0	0	0	1	1	1	0	0
0	0	0	0	0	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

10	10	10	10	10	10	10	10	10	10
10	13	13	13	10	10	11	10	11	10
10	13	13	13	10	10	10	11	10	10
10	13	13	13	10	10	11	10	11	10
10	10	10	10	10	10	10	10	10	10
10	11	10	10	10	18	18	18	10	10
10	10	10	11	10	18	18	18	10	10
10	10	11	10	10	18	18	18	10	10
10	11	10	11	10	10	10	10	10	10
10	10	10	10	10	10	11	10	10	10

```

10 10 10 10 10 10 10 10 10 10
10 13 13 13 10 10 11 10 11 10
10 13 13 13 10 10 10 11 10 10
10 13 13 13 10 10 11 10 11 10
10 10 10 10 10 10 10 10 10 10
10 11 10 10 10 18 18 18 10 10
10 10 10 11 10 18 18 18 10 10
10 10 11 10 10 18 18 18 10 10
10 11 10 11 10 10 10 10 10 10
10 10 10 10 10 10 11 10 10 10

```

peaks & valleys

```

0 0 0 0 0 0 0 0 0 0
0 1 1 1 0 0 0 0 0 0
0 1 1 1 0 0 0 0 0 0
0 1 1 1 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 1 1 0 0
0 0 0 0 0 1 1 1 0 0
0 0 0 0 0 1 1 1 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0

```

```

0 0 0 0 0 0 0 0 0 0
0 1 1 1 0 0 1 0 1 0
0 1 1 1 0 0 0 1 0 0
0 1 1 1 0 0 1 0 1 0
0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 1 1 1 0 0
0 0 0 1 0 1 1 1 0 0
0 0 1 0 0 1 1 1 0 0
0 1 0 1 0 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0

```

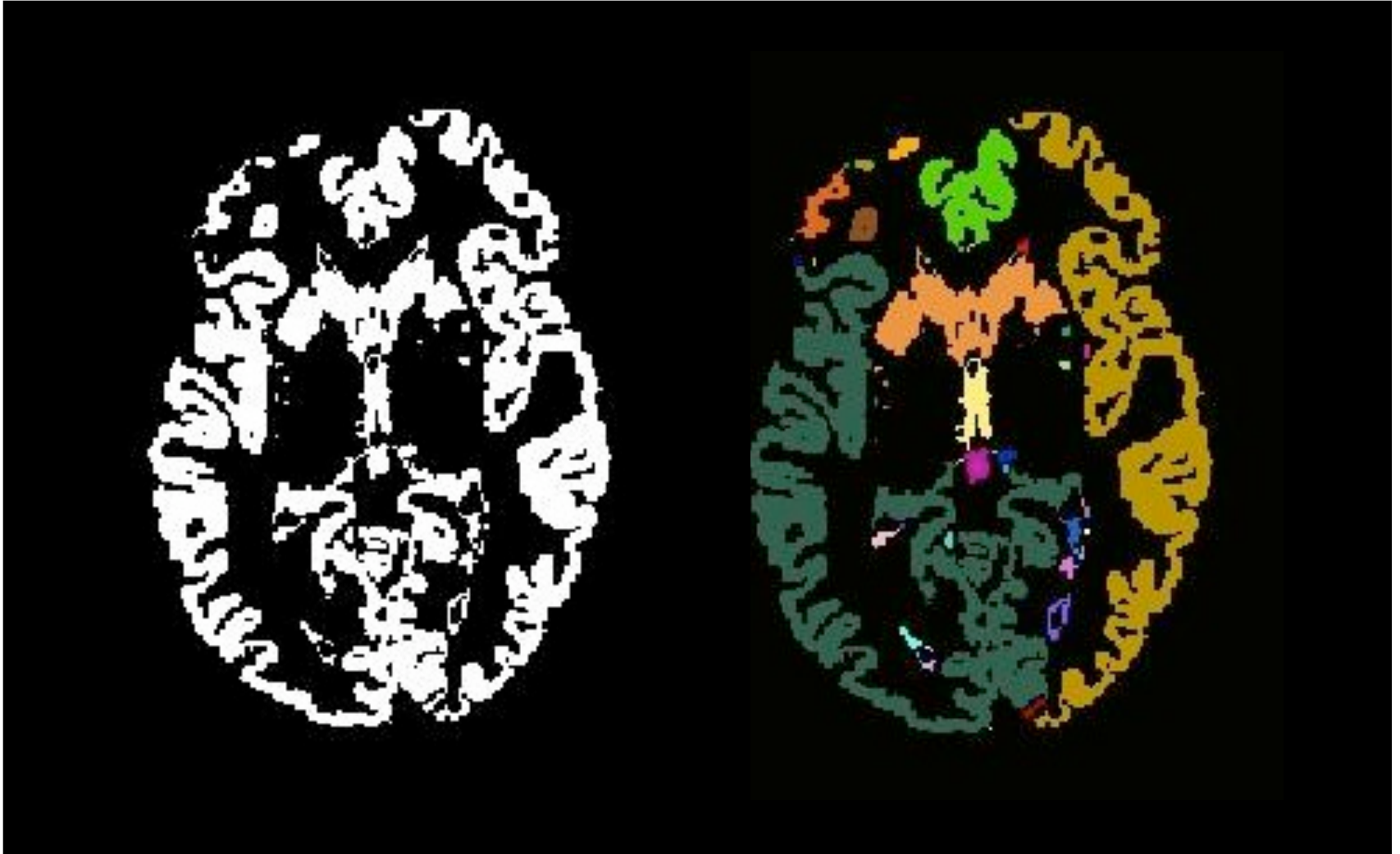
10	10	10	10	10	10	10	10	10	10
10	13	13	13	10	10	11	10	11	10
10	13	13	13	10	10	10	11	10	10
10	13	13	13	10	10	11	10	11	10
10	10	10	10	10	10	10	10	10	10
10	11	10	10	10	18	18	18	10	10
10	10	10	11	10	18	18	18	10	10
10	10	11	10	10	18	18	18	10	10
10	11	10	11	10	10	10	10	10	10
10	10	10	10	10	10	11	10	10	10

peaks & valleys

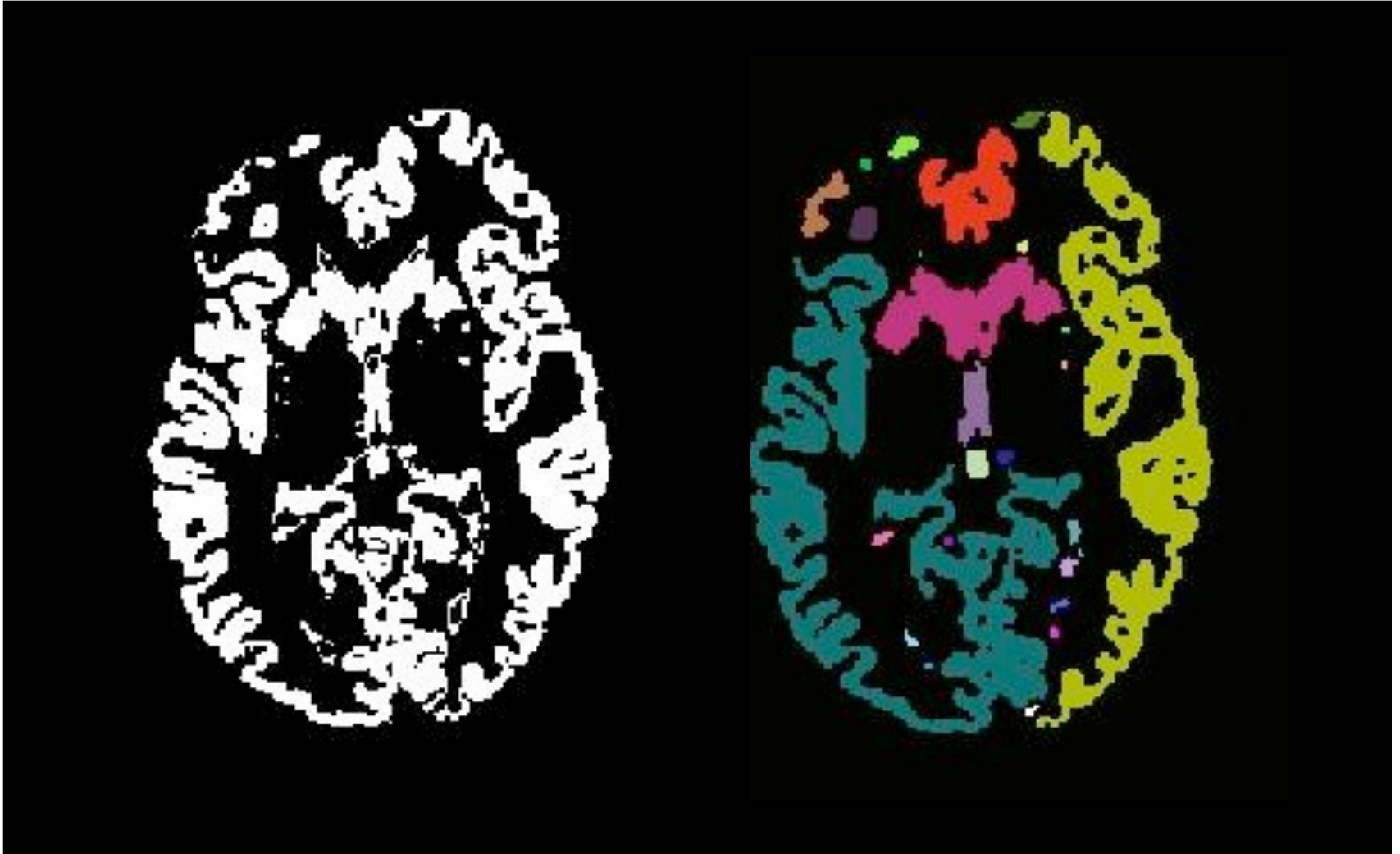
0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0
0	0	0	0	0	1	1	1	0	0
0	0	0	0	0	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0
0	0	0	0	0	1	1	1	0	0
0	0	0	0	0	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

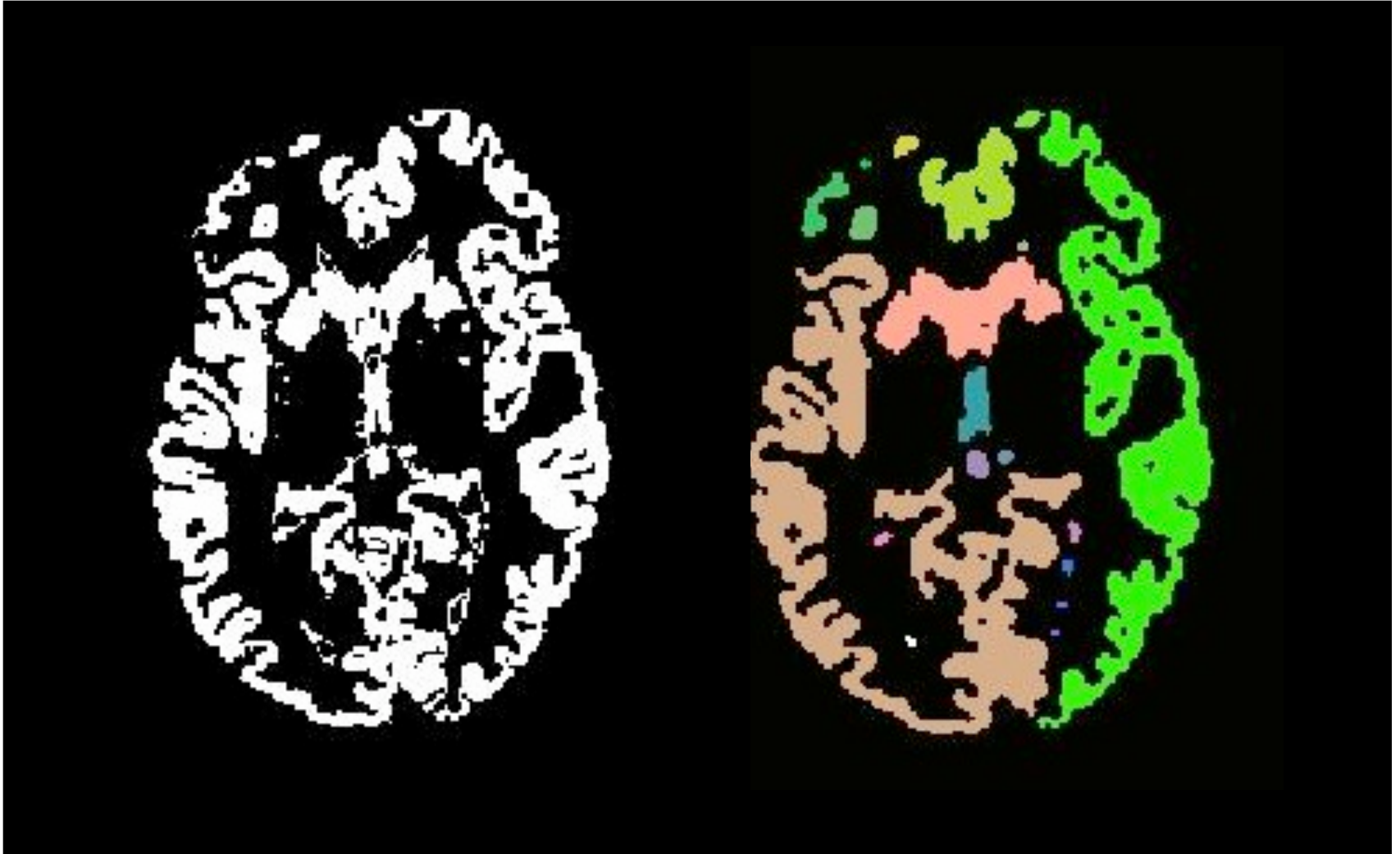
labeling connected objects



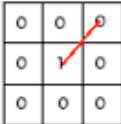
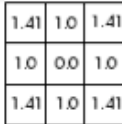
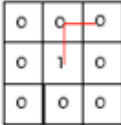

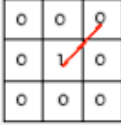
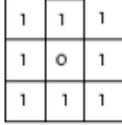
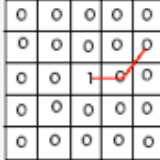

labeling connected objects



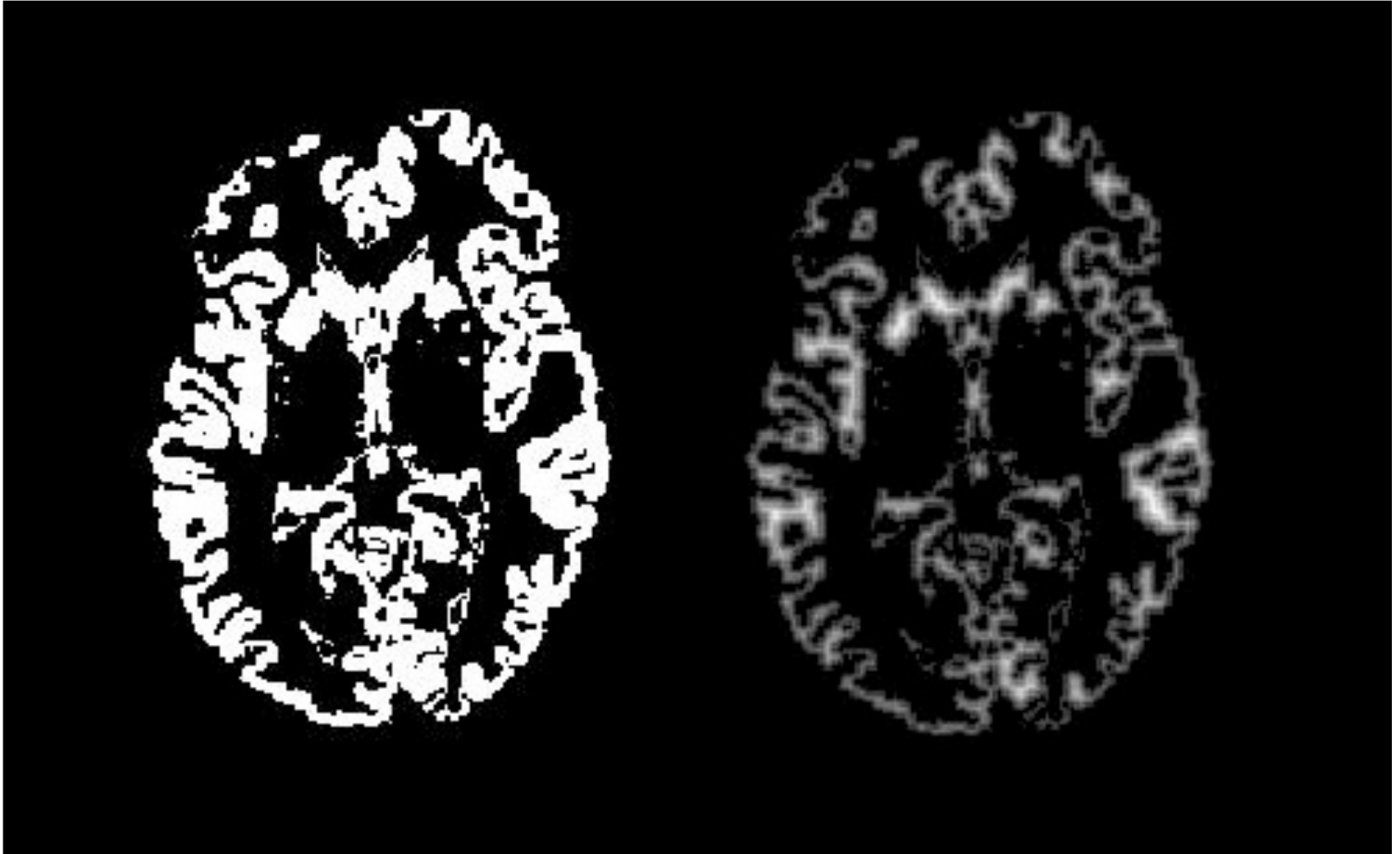
labeling connected objects



distance transforms

Distance Metric	Description	Illustration
Euclidean	The Euclidean distance is the straight-line distance between two pixels.	 Image  Distance transform
City Block	The city block distance metric measures the path between the pixels based on a 4-connected neighborhood. Pixels whose edges touch are 1 unit apart; pixels diagonally touching are 2 units apart.	 Image  Distance transform
Chessboard	The chessboard distance metric measures the path between the pixels based on an 8-connected neighborhood. Pixels whose edges or corners touch are 1 unit apart.	 Image  Distance transform
Quasi-Euclidean	The quasi-Euclidean metric measures the total Euclidean distance along a set of horizontal, vertical, and diagonal line segments.	 Image  Distance transform

distance transforms



distance transforms

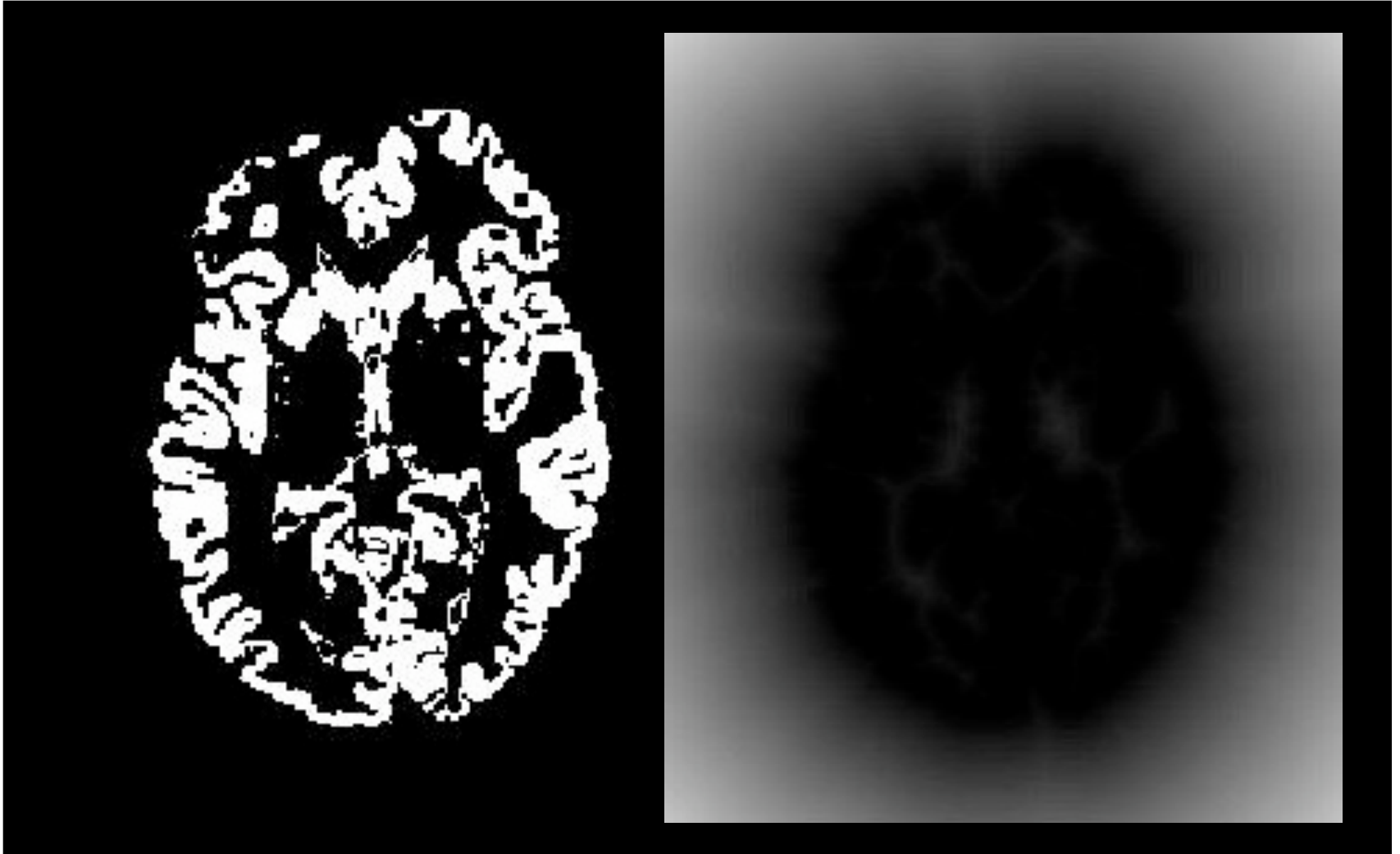
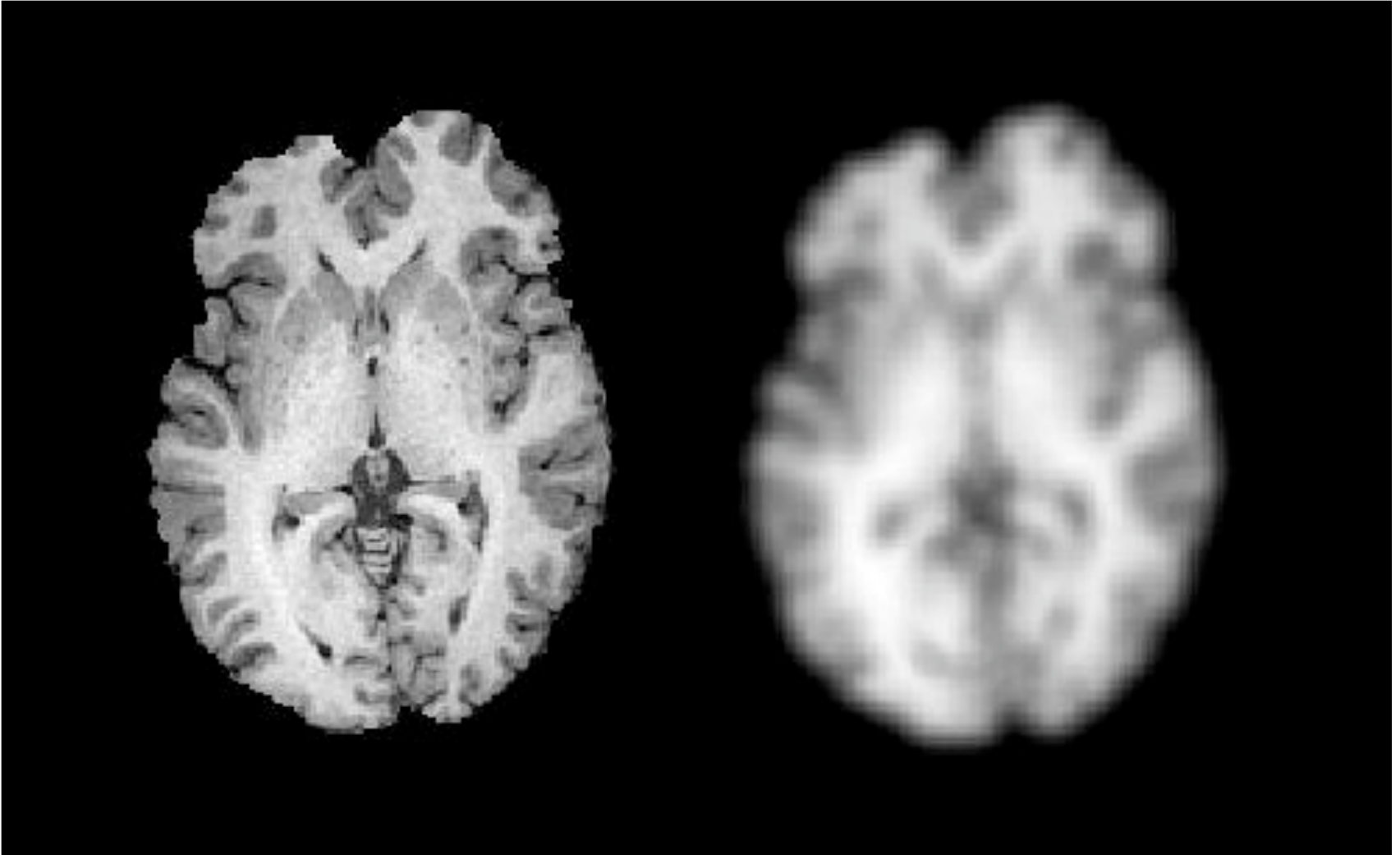


image filters

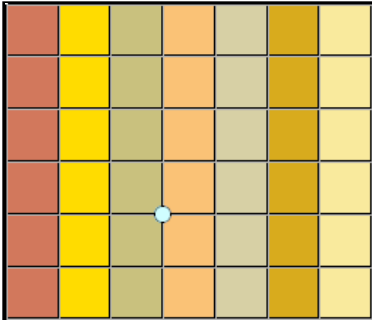


transformation matrices

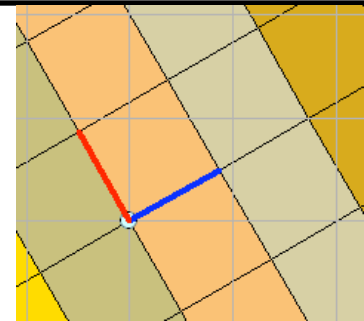
Scale

For scaling (that is, enlarging or shrinking), we have $x' = s_x \cdot x$
and $y' = s_y \cdot y$. The matrix form is:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



transformation matrices



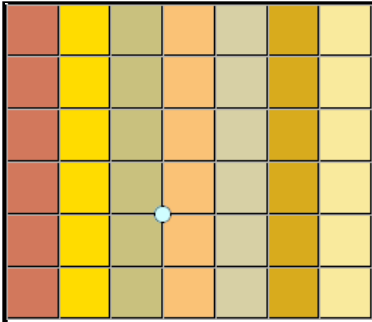
Rotation

For rotation by an angle θ counterclockwise about the origin, the functional form is $x' = x\cos\theta - y\sin\theta$ and $y' = x\sin\theta + y\cos\theta$. Written in matrix form, this becomes:

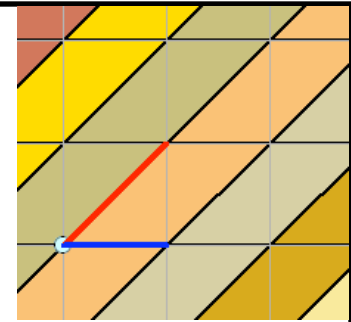
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Similarly, for a rotation clockwise about the origin, the functional form is $x' = x\cos\theta + y\sin\theta$ and $y' = -x\sin\theta + y\cos\theta$ and the matrix form is:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



transformation matrices



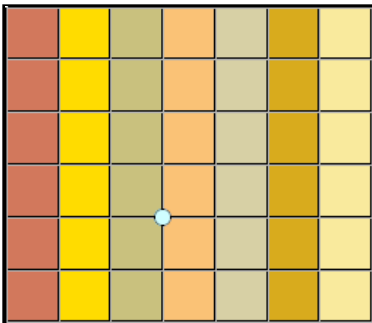
Shear

For shear mapping (visually similar to slanting), there are two possibilities. For a shear parallel to the x axis has $x' = x + ky$ and $y' = y$; the shear matrix, applied to column vectors, is:

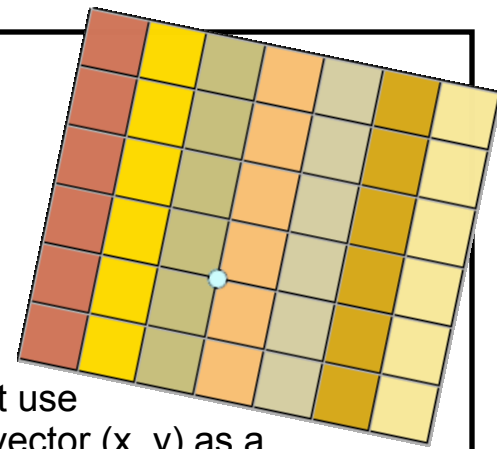
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

A shear parallel to the y axis has $x' = x$ and $y' = y + kx$, which has matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



transformation matrices



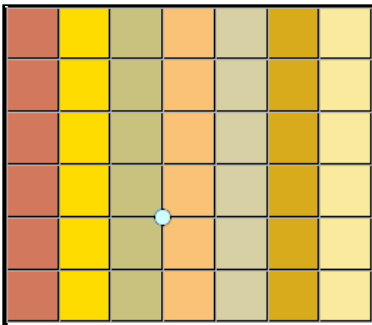
Affine: Translation

To represent affine transformations with matrices, we must use homogeneous coordinates. This means representing a 2-vector (x, y) as a 3-vector $(x, y, 1)$. The functional form $x' = x + t_x$; $y' = y + t_y$ of translation is:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} .$$

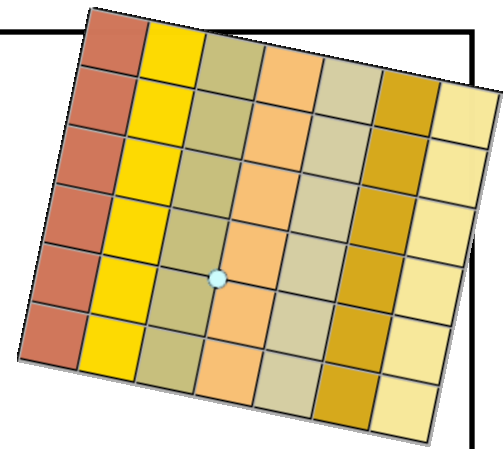
Affine: Rotation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} .$$



transformation matrices

degrees of freedom



6 dof: 3 translations, 3 rotations (x,y,z) “rigid-body”

7 dof: 3 translations, 3 rotations, 1 global scale

9 dof: 3 translations, 3 rotations, 3 scales

12 dof: 3 translations, 3 rotations, 3 scales, 3 shears

registration

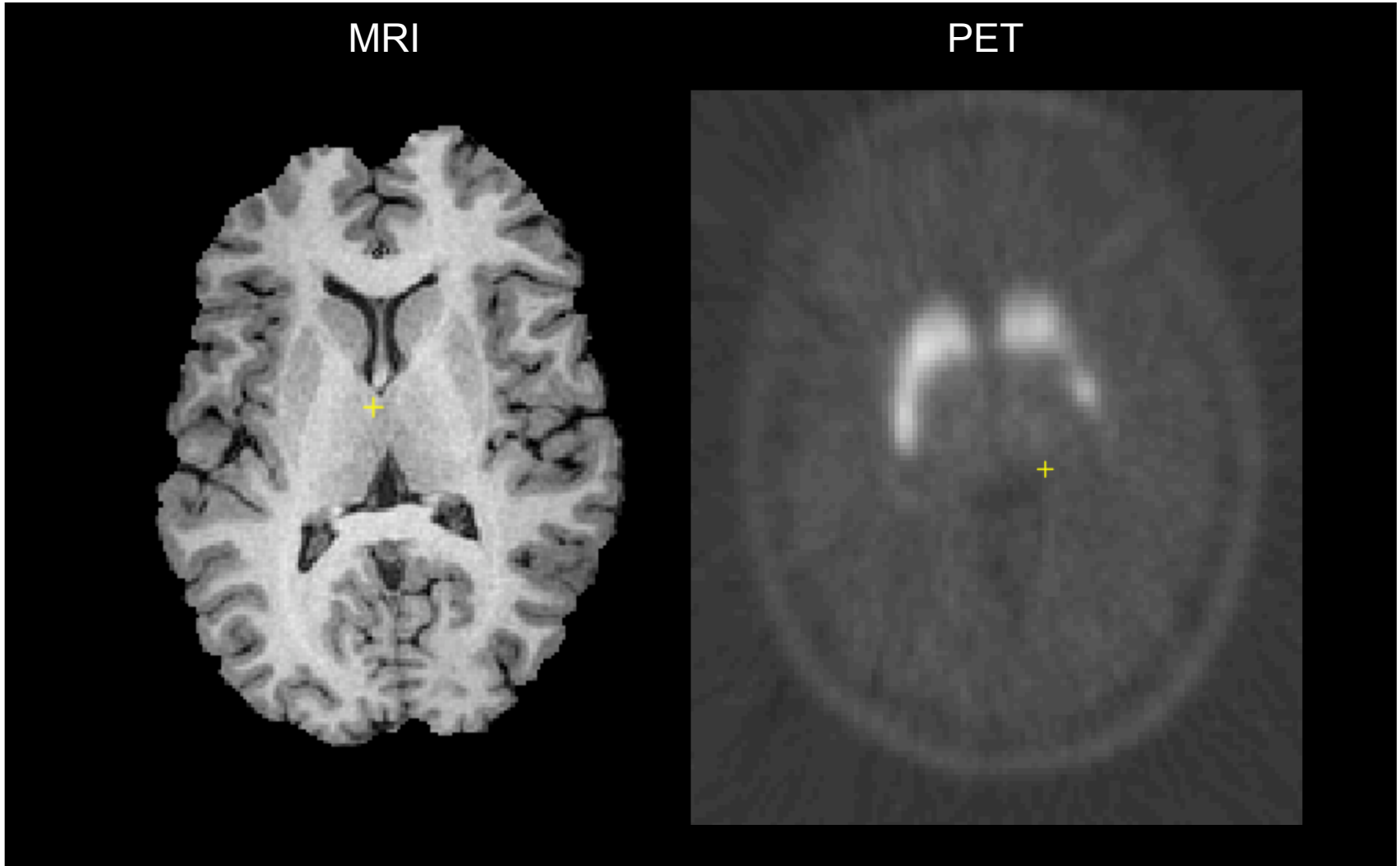
intramodality (T1-to-T1, PET-to-PET,...)

- motion correction
- group analysis
- comparative morphometry
- labeling

intermodality (T2-to-T1, PET-to-T1,...)

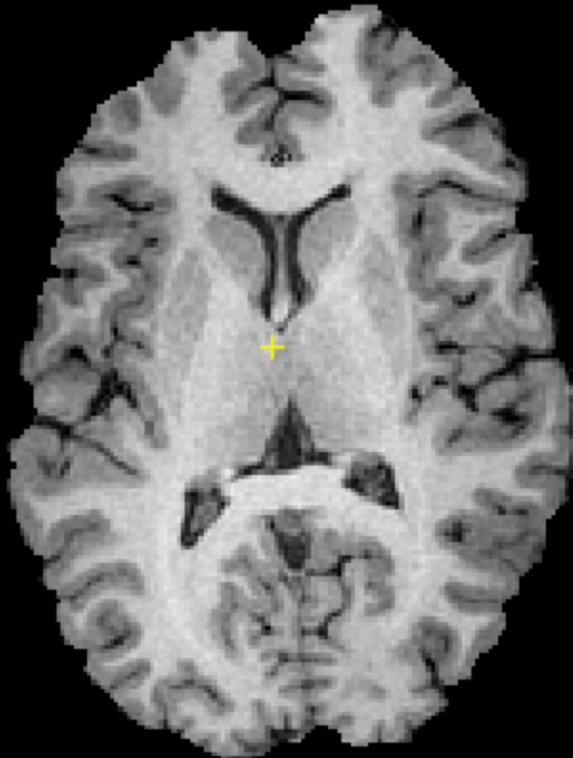
- group analysis
- multispectral analysis
- labeling

registration

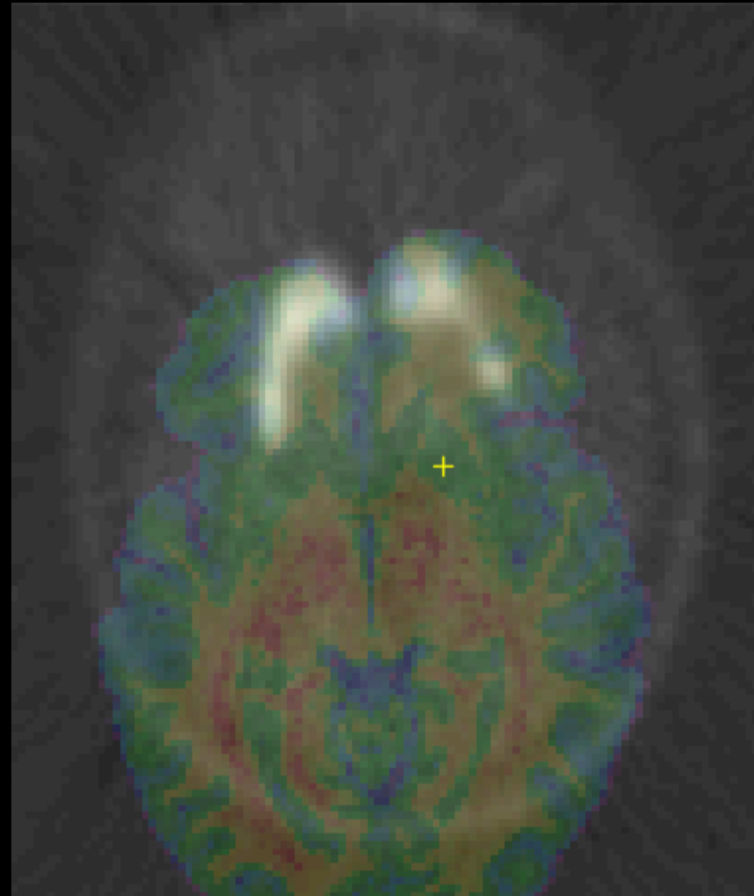


registration

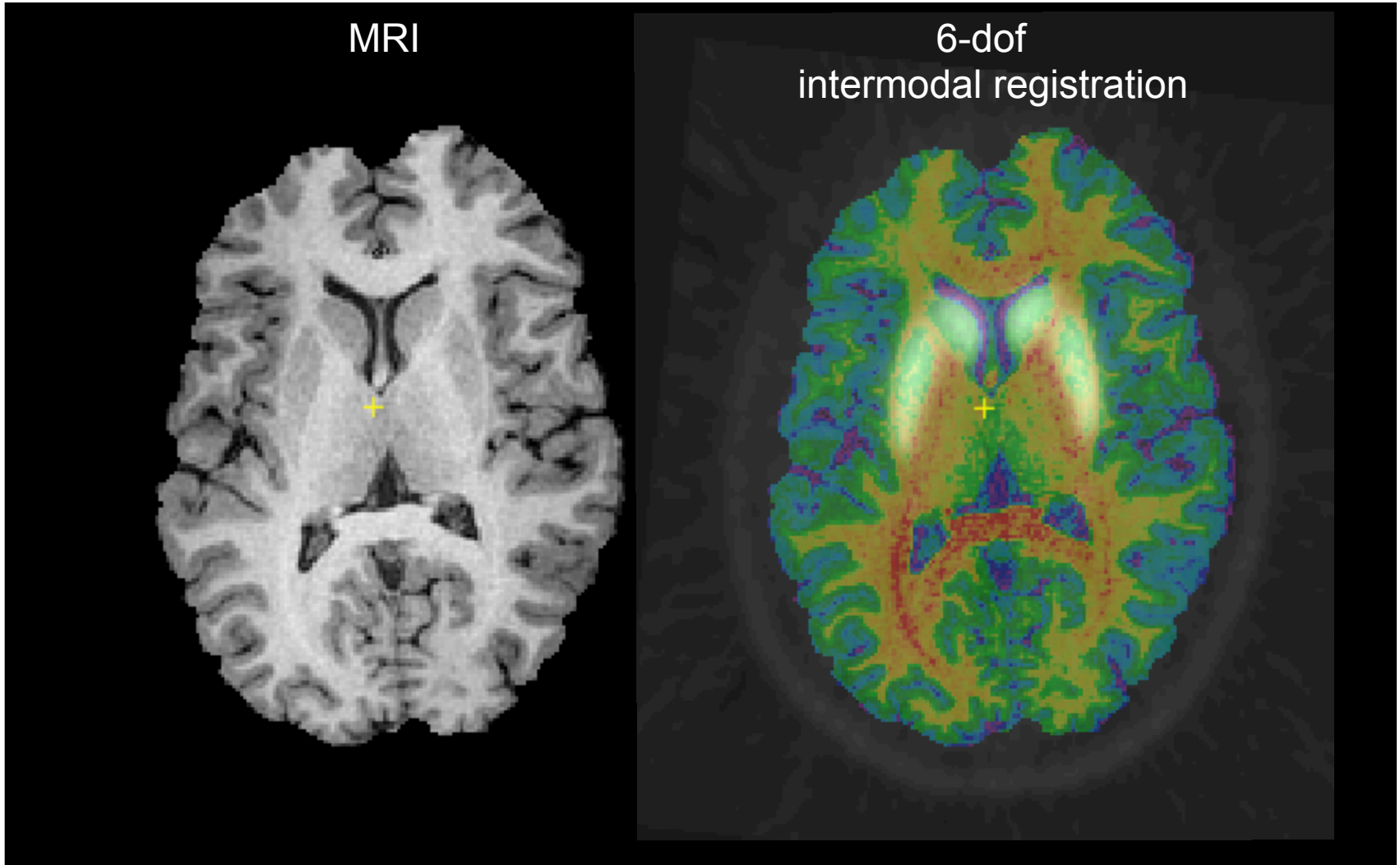
MRI



PET & MRI



registration



registration

Registration involves:

1. **Similarity** metric: (n)CR, SSD, MSD, (n)CC, MI,...
2. **Transformation** model: affine, piecewise linear, nonlinear,...
3. **Regularization** method: multi-resolution/scale, Gaussian blur,...
4. **Optimization** strategy: simplex, gradient descent,...
5. **Interpolation** type: nearest-neighbor, trilinear, cubic, sinc,...

References

<http://www.mathworks.com> (Matlab Image Processing Toolbox documentation)

<http://mathworld.wolfram.com/AffineTransformation.html>

<http://www.wikipedia.org>

<http://www.quantdec.com/GIS/affine.htm>